APPENDIX D

BASIC ENGINEERING CALCULATIONS

D-1 INTRODUCTION

This appendix consists of summaries of general areas of engineering that support topics presented in the main text or have general application to field engineering practice. Many of the equations and functions discussed can be performed by preprogrammed hand-held calculators or microcomputer-based software.

D-2 MATHEMATICS

The following paragraphs review some basic algebra, trigonometry, analytical geometry, and calculus operations. See Appendix C of the *U.S. Navy Ship Salvage Manual, Volume 1* (S0300-A6-MAN-010) or Chapter 8 of the *Salvor's Handbook* (S0300-A7-HBK-010) for mensuration of plane shapes and solid bodies.

D-2.1 Quadratic Equations. Given a quadratic equation $ax^2 + bx + c = 0$, the following relationships exist for the roots x_1 and x_2:

$$x_1,\ x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x_1 + x_2 = -\frac{b}{a}, \qquad \text{and} \quad x_1(x_2) = \frac{c}{a}$$

D-2.2 Cubic Equations. Cubic and higher order equations occur infrequently in most engineering problems, but usually are difficult to factor when they do occur. Trial and error solutions can define the general region in which a root occurs, but are generally too time consuming for precise determination of roots. Graphical means can approximate roots with fair accuracy.

Numerical analysis techniques yield extremely accurate solutions. The more efficient numerical analysis techniques are too complicated to present here. However, the bisection method described below is simple and usually can provide solutions with sufficient accuracy with only a few iterations. To use the method, values of the independent variable above and below a root, designated L_o and R_o, must be determined. The function has a value of zero at a root, so $f(L_o)$ and $f(R_o)$ have opposite signs. Let n be the iteration number. For n = 0, 1, 2, ... the following steps are iterated until sufficient accuracy is attained:

a. Set $m = 1/2\ (L_n + R_n)$.

b. Calculate $f(m)$.

c. If $f(L_n)f(m) \leq 0$, set $L_{n+1} = L_n$ and $R_{n+1} = m$; if $f(L_n)f(m) > 0$, set $L_{n+1} = m$ and $R_{n+1} = R_n$.

$f(x)$ has at least one root in the interval (L_{n+1}, R_{n+1}), with an estimated value of:

$$x \approx \frac{1}{2}\left(L_{n+1} + R_{n+1}\right)$$

The maximum error is $\frac{1}{2}(R_{n+1} - L_{n+1})$. The bisection method does not automatically find other roots that may exist on the real number line.

EXAMPLE D-1

Find the roots of $f(x) = x^3 - 2x - 7$

The first step is to find L_o and R_o. The approximate vicinity of the root can be determined by calculating $f(x)$ for arbitrary values of x, as shown below:

x	-2	-1	0	+1	+2	+3
$f(x)$	-11	-6	-7	-8	-3	+14

$f(x)$ changes sign between $x = 2$ and $x = 3$, so a root exists in the interval (2, 3); set $L_o = 2$ and $R_o = 3$

Iteration 0:

$$m = \frac{1}{2}(2+3) = 2.5$$

$$f(2.5) = (2.5)^3 - 2(2.5) - 7 = 3.625$$

$f(2.5)$ is positive, so a root exists in the interval (2, 2.5); set $L_1 = 2$ and $R_1 = 2.5$. At this point, the best estimate of the root is the value m that would be used for the next iteration:

$$m_1 \approx \frac{1}{2}(2+2.5) = 2.25$$

The maximum error is 1/2(2.5 - 2) = 0.25.

Iteration 1:

$$f(m_1) = f(2.25) = -0.1094$$

$f(m)$ is negative so a root exists in the interval (2.25, 2.5); set $L_2 = 2.25$ and $R_2 = 2.5$. The best estimate of the root is:

$$x \approx \frac{1}{2}(2.25+2.5) = 2.375$$

The maximum error is 1/2(2.5 - 2.25) = 0.125. The procedure continues until the maximum error is acceptable.

D-2.3 Trigonometry. Trigonometry provides angular relationships that can be used to determine length of sides and size of included angles in triangles and polygons, and to resolve vectors into rectilinear components.

D-2.3.1 Angular Measure

$$360 \text{ degrees} = \text{one complete circle} = 2\pi \text{ radians}$$

$$90 \text{ degrees} = \text{right angle} = \frac{\pi}{2} \text{ radians}$$

$$\text{one radian} = \frac{180}{\pi} = 57.3 \text{ degrees}$$

$$\text{one degree} = \frac{\pi}{180} = 0.0175 \text{ radians}$$

D-2.3.2 Right Triangles. For the right triangle shown in Figure D-1, side h is the hypotenuse, side x is adjacent to, and side y opposite of, angle θ. The Pythagorean Theorem states that $x^2 + y^2 = h^2$, and forms the basis for the definitions of the trigonometric functions sine (sin), cosine (cos), tangent (tan), cotangent (cot), secant (sec), and cosecant (csc) shown below:

$$\sin\theta = \frac{y}{h} = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{x}{h} = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{h}{y} = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{h}{x} = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{x}{y} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

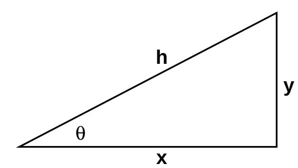

Figure D-1. Right Triangle.

Figure D-2 shows the relationship of the trigonometric functions to the unit circle. The functions of the related angles are given in Table D-1. Table D-2 of the *U.S. Navy Ship Salvage Manual, Volume 1* (S0300-A6-MAN-010) gives trigonometric functions for angles from 0 to 90 degrees.

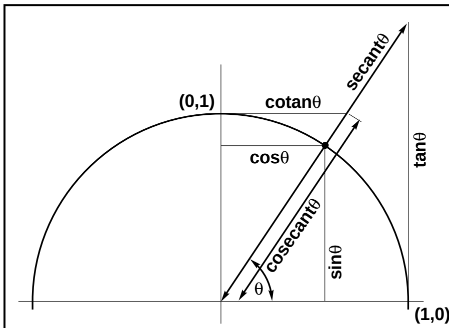

Figure D-2. Unit Circle.

Table D-1. Functions of Related Angles.

Function	Numerically equal function of theta for the indicated angle				
	-θ	**90 - θ**	**90 + θ**	**180 - θ**	**180 + θ**
sin	-sinθ	cosθ	cosθ	sinθ	-sinθ
cos	cosθ	sinθ	-sinθ	-cosθ	-cosθ
tan	-tanθ	cotθ	-cotθ	-tanθ	tanθ

D-2.3.3 Trigonometric Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \cot^2\theta + 1 = \csc^2\theta \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$\sin 2\theta = 2(\sin\theta)(\cos\theta) \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$$

$$\sin\theta = 2\left[\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] = \tan\theta\,\cos\theta$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1}{2}(1 - \cos\theta)} \qquad \tan\theta = \sin\theta\,\sec\theta \qquad \cos\theta = \cot\theta\,\sin\theta$$

$$\sec\theta = \csc\theta\,\tan\theta \qquad \cot\theta = \cos\theta\,\csc\theta \qquad \csc\theta = \sec\theta\,\cot\theta$$

D-2.3.4 Two-angle Formulae. For the two acute angles of a right triangle:

$$\theta + \phi = 90^{\circ}$$

$$\sin(\theta + \phi) = [\sin\theta][\cos\phi] + [\cos\theta][\sin\phi]$$

$$\sin(\theta - \phi) = [\sin\theta][\cos\phi] - [\cos\theta][\sin\phi]$$

$$\cos(\theta + \phi) = [\cos\theta][\cos\phi] - [\sin\theta][\sin\phi]$$

$$\cos(\theta - \phi) = [\cos\theta][\cos\phi] + [\sin\theta][\sin\phi]$$

D-2.3.5 General Triangles. For any triangle, as shown in Figure D-3, the following laws apply:

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines: $a^2 = b^2 + c^2 - 2bc(\cos A)$

Area: $\frac{1}{2}ab(\sin C)$

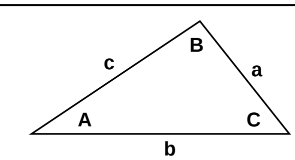

Figure D-3. General Triangle.

D-2.3.6 Hyperbolic Functions. Hyperbolic functions are specific equations that include the terms e^x and e^{-x}. These combinations of e^x and e^{-x} appear regularly in certain types of problems. In order to simplify the equations in which they appear, hyperbolic functions are given trigonometric names and symbols:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$$

$$\operatorname{csch} x = \frac{2}{e^x - e^{-x}} = \frac{1}{\sinh x} \qquad \operatorname{sech} x = \frac{2}{e^x + e^{-x}} = \frac{1}{\cosh x} \qquad \coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{\cosh x}{\sinh x}$$

The hyperbolic identities differ somewhat from the standard trigonometric identities. Several of the most common identities are shown here:

$$\cosh^2 x - \sinh^2 x = 1 \qquad 1 - \coth^2 x = \operatorname{csch}^2 x \qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\cosh x + \sinh x = e^x \qquad \cosh x - \sinh x = e^{-x}$$

$$\cosh(x + y) = (\cosh x)(\cosh y) + (\sinh x)(\sinh y) \qquad \sinh(x + y) = (\sinh x)(\cosh y) + (\cosh x)(\sinh y)$$

D-2.4 Straight-line Analytic Geometry. For the straight line shown in Figure D-4, the following equations can be written:

General Form: $ax + by + c = 0$

Slope Form: $y = mx + b$

Point-slope Form: $(y - y_o) = m(x - x_o)$

where (x_o, y_o) is any point on the line

Intercept Form: $\frac{x}{a} + \frac{y}{b} = 1$

Two-point Form: $\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2}$

Polar Form: $r\sin\phi = d_1$

Normal Form: $x(\cos\phi) + y(\sin\phi) = d_1$

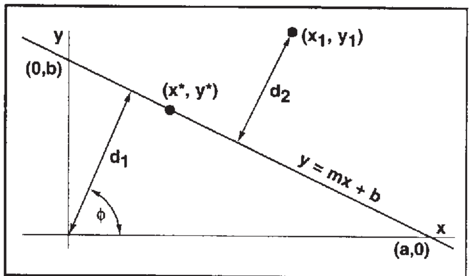

Figure D-4. Straight Line.

The distance d between a point and a line is:

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

The distance between two points on a rectangular coordinate system is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For parallel lines:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}, \qquad m_1 = m_2$$

For perpendicular lines:

$$a_1a_2 = b_1b_2, \qquad m_1 = -\frac{1}{m_2}$$

Point of intersection of two lines:

$$x_1 = \frac{b_2c_1 - b_1c_2}{a_2b_1 - a_1b_2}, \qquad y_1 = \frac{a_1c_2 - a_2c_1}{a_2b_1 - a_1b_2}$$

Smaller angle between two intersecting lines:

$$\tan\phi = \frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2} = \frac{m_2 - m_1}{1 + m_1m_2}, \qquad \phi = |\arctan m_1 - \arctan m_2|$$

D-2.5 Differential Calculus. Derivatives can be used to locate local maxima, minima, and points of inflection. No distinction is made between local and global extrema. The end points of the interval always should be checked against the local extrema located by derivatives. The following rules define the extreme points:

$f'(x) = 0$ at any extrema

$f''(x) = 0$ at an inflection point

$f''(x) < 0$ at a maximum

$f''(x) > 0$ at a minimum

With a few special exceptions (i.e., some trigonometric functions), there is always an inflection point between a maximum and a minimum.

D-2.6 Integral Calculus. The Fundamental Theorem of Calculus is:

$$\int_{x_1}^{x_2} f'(x) = f(x_2) - f(x_1)$$

D-2.6.1 Integration by Parts. If f and g are functions, then:

$$\int f\, dg = fg - \int g\, df$$

D-2.6.2 Indefinite Integrals. The following list includes some of the more common indefinite integrals. u and v are functions of the variable x; a is a constant. The constant terms ("...+ C") have been omitted from the solved integrals.

$$\int dx = x$$

$$\int a dx = ax$$

$$\int au\, dx = a\int u\, dx$$

$$\int u dv = uv - \int v du$$

$$\int (u+v) dx = \int u\, dx + \int v\, dx$$

$$\int dy \int f(x,y) dx = \int dx \int f(x,y) dy$$

$$\int x^m dx = \frac{x^{(m+1)}}{m+1} \qquad m \neq -1$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int e^x dx = e^x$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int x e^{ax} dx = \frac{1}{a^2} e^{ax} (ax - 1)$$

$$\int \frac{dx}{1+x^2} = \arctan x$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \operatorname{arcsec} x$$

$$\int \cos x\, dx = \sin x$$

$$\int (\sin ax) dx = \frac{-1}{a} \cos ax$$

$$\int \sin x\, dx = -\cos x$$

$$\int (\cos ax) dx = \frac{1}{a} \cos ax$$

$$\int \tan x\, dx = \ln|\sec x| = -\ln|\cos x|$$

$$\int (\tan ax) dx = \frac{1}{a} \log \sec ax = -\frac{1}{a} \log \cos ax$$

$$\int \cot x\, dx = \ln|\sin x| = -\ln|\csc x|$$

$$\int (\cot ax) dx = \frac{1}{a} \log \sin ax = -\frac{1}{a} \log \csc ax$$

$$\int \sec x\, dx = \ln|\sec x + \tan x|$$

$$\int (\sec ax) = \frac{1}{a} \log(\sec ax + \tan ax) = \frac{1}{a} \log \tan\left(\frac{\pi}{4} + \frac{2x}{2}\right)$$

$$\int \csc x\, dx = \ln|\csc x + \cot x|$$

$$\int (\csc ax) dx = \frac{1}{a} \log(\csc ax - \cot ax) = \frac{1}{a} \log \tan \frac{ax}{2}$$

$$\int \cosh x\, dx = \sinh x$$

$$\int \sinh x\, dx = \cosh x$$

D-2.6.3 Integral Tables. It is not uncommon that an integral to be evaluated is not found in the integral table in use, especially with an abbreviated table like the one above. When this occurs, there are three basic alternatives:

- Seek a more extensive integral table, such as those contained in the *CRC Standard Mathematical Tables*, *Burrington's Math Tables*, *Mark's Standard Handbook for Mechanical Engineers*, or various calculus texts, such as Thomas' *Calculus and Analytical Geometry*.

- Apply numerical or approximate methods (see Paragraph 1-4).

- Attempt to transform the integral into a form that can be evaluated. Some brief guidance on transforming integrals follows. For a more complete discussion, consult a standard calculus text, such as Thomas' *Calculus and Analytical Geometry*.

Expressions to be integrated, or *integrands*, are transformed by four basic methods:

- *Trigonometric Substitutions* – If an integrand contains $\sqrt{(a^2 - x^2)}$, substitute $a\sin u$ for x. $\sqrt{(a^2 - x^2)}$ then becomes $a\cos u$. Similarly, substitute $a\tan u$ for x, and $a\sec u$ for $\sqrt{(x^2 + a^2)}$, or $a\sec u$ for x, and $a\tan u$ for $\sqrt{(x^2 - a^2)}$

- *Completing the Square* – Rewrite $ax^2 + bx + c$ as $a[x + b/2a]^2 + (4ac - b^2)/4a$ and substitute $u = x + b/2a$ and $B = (4ac - b^2)/4a$.

- *Partial Fractions* – For a ratio of polynomials, where the denominator has been factored into linear factors $p_i(x)$ and quadratic factors $q_j(x)$, and the degree of the numerator $r(x)$ is less than that of the denominator, rewrite $r(x)/[p_1(x) \ldots p_n(x)q_1(x) \ldots q_m(x)]$ as $A_1/p_1(x) + \ldots + A_n/p_n(x) + (B_1x + C_1)/q_1(x) + \ldots + (B_mx + C_m)/q_m(x)$.

- *Integration by Parts* – Change the integral using the formula $\int u\,dv = uv - \int v\,du$, where u and dv are chosen so that v is easy to find from dv, and $v\,du$ is easier to find than $u\,dv$.

D-2.6.4 Uses of Integrals. The principal uses of integration are for determining areas and volumes of shapes bounded by continuous curves.

The area bounded by $x = a$, $x = b$, $f_1(x)$ above, and $f_2(x)$ below is given by:

$$\text{Area} = \int_a^b [f_1(x) - f_2(x)]\,dx$$

The area of the surface created by rotating a function $f(x)$ about the X-axis is:

$$\text{Surface area} = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}\;dx$$

The volume of a function rotated about the X-axis is:

$$\text{Volume} = \pi \int_a^b (f(x))^2\,dx$$

The volume of a function rotated about the Y-axis is:

$$\text{Volume} = 2\pi \int_a^b x f(x)\,dx$$

The length of a curve described by $f(x)$ is:

$$\text{Length} = \int_a^b \sqrt{1 + [f'(x)]^2}\;dx$$

D-2.7 Miscellaneous Constants

$\sqrt{2} = 1.141421$	$\pi = 3.14159$	$e = 2.71828$
$\sqrt[3]{2} = 1.25992$	$\dfrac{1}{\pi} = 0.31830$	$\dfrac{1}{e} = 0.36787$
$\ln 2 = 0.69314$	$\pi^2 = 9.86960$	$e^2 = 7.38905$
$\sqrt{3} = 1.73205$	$\ln \pi = 1.14472$	$\log_{10} e = 0.43429$
$\sqrt[3]{3} = 1.44224$	$\log_{10} \pi = 0.49714$	$\ln 10 = 2.30258$
$\ln 3 = 1.09861$		

D-3 SOLID MECHANICS

Mechanics is the branch of science that deals with forces and motion. *Statics* deals with the action of forces on bodies at rest, while *dynamics* deals with forces on bodies in motion.

D-3.1 Units of Mass and Force. Units of mass and force are often confused, particularly in the English gravitational (foot-pound-second) system of units. Mass is a measure of physical quantity; the mass of an object is independent of gravity or other acceleration. Force is related to mass by Newton's second law of motion:

$$F = \frac{d}{dt}mv = m\frac{dv}{dt} + v\frac{dm}{dt}$$

where m is the mass and v the velocity of the body in question, and d/dt indicates rate of change (derivative with respect to time) of the designated property. Mass is constant for most situations, so dm/dt is zero, and change of velocity with respect to time is acceleration (a) giving Newton's second law in its familiar form:

$$F = ma$$

In the English system, the pound is commonly used both as a unit of force (pound-*force*, lbf) and as a unit of mass (pound-*mass*, lbm). If these units are substituted into Newton's second law, with acceleration in feet per second per second (ft/sec^2), the units do not balance:

$$\text{lbf} = (\text{lbm})(\text{ft/sec}^2)$$

This discrepancy is resolved by multiplying the equation by a constant with appropriate units:

$$\text{lbf} = (\text{lbm})\left(\frac{\text{ft}}{\text{sec}^2}\right) \times C\ \frac{\text{lbf-sec}^2}{\text{lbm-ft}}$$

One pound force is defined as the force exerted by a standard gravitational field on one pound mass. Standard acceleration due to gravity, g_o, is taken as 32.174 ft/sec^2. Substituting these values and units into the force equation and solving for C:

$$1\ \text{lbf} = (1\ \text{lbm})(32.1739\ \text{ft/sec}^2) \times C\ \text{lbf-sec}^2/\text{lbm-ft}$$

$$\therefore\quad C\ \text{lbf-sec}^2/\text{lbm-ft} = \frac{1\ \text{lbf}}{32.174\ \text{lbm-ft/sec}^2}$$

The force equation can now be written:

$$F = Cma = \frac{ma}{g_c}$$

where g_c is the gravitational acceleration constant, numerically equal to the standard acceleration due to gravity (32.174), with units of lbm-ft/lbf-sec^2. The *slug* is defined as the mass that will be accelerated at 1 ft/sec^2 by a force of 1 lbf. A slug is therefore equal to 32.174 lbm. If mass is expressed in slugs, g_c is 1 slug-ft/lbf-sec^2.

In the International System of Units (SI), the unit of force is the newton (N), defined as the force required to accelerate a mass of kilogram at one meter per second per second. Substitution into the force equation shows that with mass expressed in kilograms, g_c is 1 kg-m/N-sec^2. A metric gravitational system, with force units of kilograms-***force*** (kgf), is sometimes used, although it is gradually being displaced by the more correct SI *newton*. Again, substitution of units into the force equation shows that to obtain forces in kilograms-force, g_c must have a value of 9.80655 kg-m/kgf-sec^2.

The constant g_c is always present in the force equation, and consequently in many equations derived from it. Deletion of g_c or confusion of g_c with the acceleration of gravity (g) are common errors in applying these equations. Depending on the units used, these errors may have no practical consequence. The engineer should understand the function of g_c in the relationships in use, and its numerical value relative to the desired units, to avoid serious calculation errors.

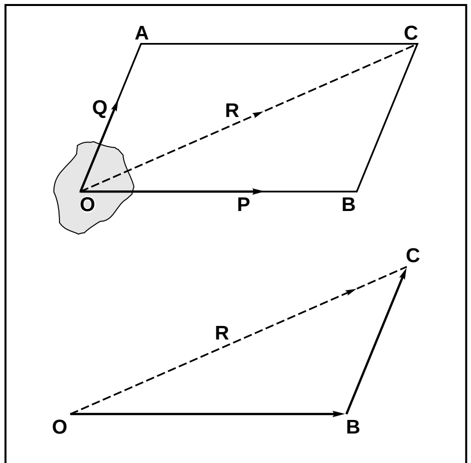

Figure D-5. Resultants of Coplanar Concurrent Forces.

D-3.2 Statics. *Forces* tend to change the state of rest or motion of a body. A force is completely specified by its magnitude, direction, and point of application. The word *sense* as applied to a force refers to one of the two directions along the line of action of the force. Forces are represented graphically by vectors with direction parallel to the forces line of action and lengths proportional to the magnitude of the force. A drawing showing the lines of action of forces acting on a body or structure is a *space diagram*. A sketch showing vectors representing the forces is a *vector diagram*. In the following discussions, forces are indicated on space diagrams by two lowercase letters next to their lines of action; corresponding force vectors in the vector diagram are identified by the same uppercase letters marking the endpoints. The sequence of letters identifying a vector indicates the sense of the vector; vector *BA* is equal to, but with opposite sense of vector *AB*. A number of forces taken collectively is a system or set of forces. Force systems are classified as *coplanar*, with the lines of action of all forces lying in the same plane, or *non*coplanar. Force systems are further classified as *concurrent, nonconcurrent,* or *parallel*, depending on whether all the forces intersect at a single point, intersect at several points, or have parallel lines of action. Two or more forces that are equivalent to a single force are *components* of the single force. *Composition of forces* is the replacing of a system of forces with a simpler system. *Resolution of forces* is the replacing of a single force by a system of forces, usually with lines of action parallel to coordinate system axes. The *resultant* of a force system is the simplest equivalent system. For concurrent, coplanar forces, the resultant is a single force. For nonconcurrent or parallel coplanar forces, the resultant may be either a force or a couple. For noncoplanar forces, the resultant may be two or more forces that are not parallel and do not intersect.

D-3.2.1 Coplanar Concurrent Forces. Two concurrent forces, *P* and *Q*, acting through point *O* on a body, are represented by the adjacent sides *OB* and *OA* of parallelogram *OACB*, as shown in Figure D-5. The resultant is represented by the diagonal *OC*. Since side *BC* is equal to side *OA*, the force system can also be represented by triangle *OBC* as shown. The length of *OC* is determined by the law of sines or cosines, or the Pythagorean theorem if *OBC* is a right triangle.

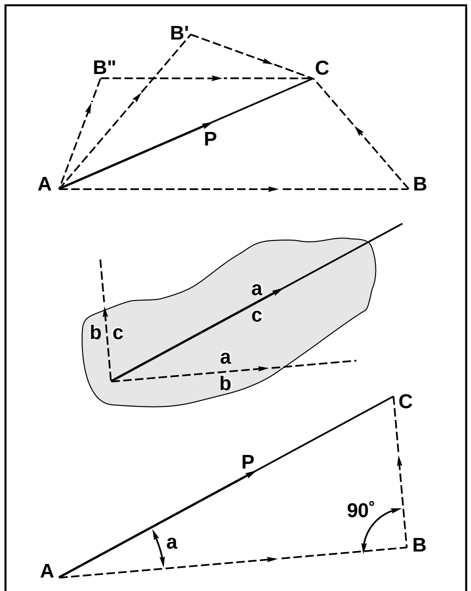

Figure D-6. Resolution of Forces.

A force can be resolved into an infinite number of pairs of components by constructing different triangles, as shown in Figure D-6. The most common task is to resolve the force into rectangular components, parallel to the axes of the chosen coordinate system. The unresolved force vector forms the hypotenuse of a right triangle, with the component forces forming the adjacent and opposite legs as shown. Given angle α, $P_x = P\cos\alpha$ and $P_y = P\sin\alpha$ for the force triangle shown in Figure D-6.

The resultant of multiple concurrent forces can be found graphically by constructing a force polygon, as shown in Figure D-7. The resultant can be determined algebraically by resolving each force into *x* and *y* rectangular components, with components acting upwards or to the right as positive, and those acting downwards or to the left as negative. The *x* and *y* components are summed separately, and the results recombined to form the resultant of the force system.

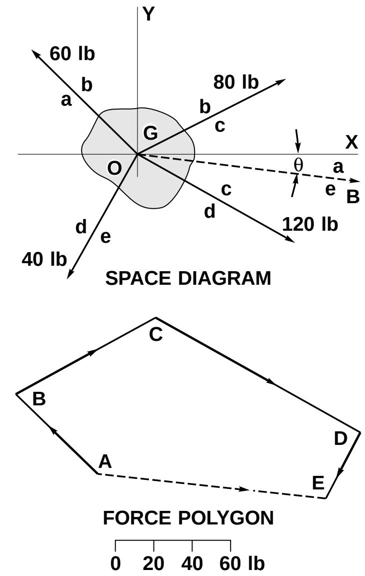

Figure D-7. Force Polygon.

D-3.2.2 Noncoplanar Concurrent Forces. The resultant of three rectangular noncoplanar concurrent forces *P*, *Q*, and *S* is determined by constructing a parallelopiped, as shown in Figure D-8. The resultant is represented by the diagonal *R*, with magnitude $\sqrt{(P^2 + Q^2 + S^2)}$. Its direction cosines with respect to the axes are given by:

$$\cos\alpha = \frac{P}{R}, \quad \cos\beta = \frac{Q}{R}, \quad \cos\gamma = \frac{S}{R}$$

The resultant of any number of noncoplanar, nonrectangular concurrent forces can be determined if the forces are specified with reference to three rectangular axes passing through the point of currency. Each force is resolved into *x*, *y*, and *z* components that are summed and recombined into the resultant. If the component summations are designated ΣF_x, ΣF_y, and ΣF_z, then:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

$$\phi_x = \arccos\frac{\Sigma F_x}{R} \qquad \phi_y = \arccos\frac{\Sigma F_y}{R} \qquad \phi_z = \arccos\frac{\Sigma F_z}{R}$$

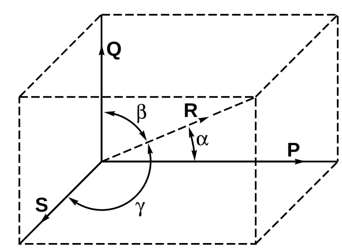

Figure D-8. Resultant of Noncoplanar Force System.

D-3.2.3 Moments and Couples. The *moment*, or *torque*, of a force about a point is the product of the force magnitude and the moment arm (the distance separating the force line of action and the point). When working with moments, it is important to adopt and maintain a consistent sign convention.

Generally, moments tending to produce counter-clockwise rotation are taken as positive.

The moment of a force about a particular axis is determined by resolving the force into components parallel with and perpendicular to the axis. The parallel component produces no moment. The sum of the moments of any coplanar force system about any point or axis in their plane is equal to the moment of the resultant about the same point or axis.

Two equal and parallel forces of opposite sense form a *couple*. The arm of the couple is the distance between the lines of action. The moment of a couple is equal to the product of the magnitude of one of the forces and the arm of the couple; the moment is constant and independent of the origin of the moments. Couples of equal moments, in the same or parallel planes, are equivalent, and may replace one another—a couple may be rotated or moved in its own plane, or transferred to any parallel plane, without altering the resulting motion of the body on which it acts. A couple may be represented by a vector length equal to the magnitude of the couple's moment. The vector is drawn perpendicular to the plane of the couple. The positive sense of the vector is the direction in which a right-hand screw would advance. The resultant of coplanar couples, or couples in parallel planes, is a couple with moment and sense equal to the algebraic sum of the component couples. The resultant of any number of couples in oblique or parallel planes is also a couple. The resultant is determined by resolving each couple vector into components parallel to a set of rectangular axes. The *x*, *y*, and *z* components are summed as ΣC_x, ΣC_y, and ΣC_z; the magnitude and direction angles of the resultant couple vector *C* are given by:

$$C = \sqrt{(\Sigma C_x)^2 + (\Sigma C_y)^2 + (\Sigma C_z)^2}$$

$$\phi_x = \arccos\frac{\Sigma C_x}{C} \qquad \phi_y = \arccos\frac{\Sigma C_y}{C} \qquad \phi_z = \arccos\frac{\Sigma C_z}{C}$$

D-3.2.4 Coplanar Nonconcurrent Forces. The resultant of a system of coplanar nonconcurrent forces may be either a single force or a couple. Resultants are found graphically by use of a *funicular* diagram, a special type of vector diagram drawn next to the space diagram. For example, to find the resultant of the four parallel forces shown in Figure D-9:

a. Plot vector *AB* to show the sense and magnitude of force *ab* parallel to the line of action at a convenient location. Then plot, in succession, the vectors *BC*, *CD*, and *DE* to show the sense and magnitude of forces *bc*, *cd*, and *de*. The vectors plot on the same line, in either an upward or downward direction, as dictated by the sense of the vector force. *AE*, *running* from the beginning to the end of the vector diagram, shows the magnitude and sense of the resultant, in this case, 180 pounds, downward.

b. To determine the line of action *ae* of vector *AE*, plot the pole *O* in any convenient position and draw the rays *AO*, *BO*, *CO*, *DO*, and *EO* as shown.

c. From any point on the line of action *ab* (in the space diagram), draw strings *ao* and *ob* parallel to *AO* and *OB*.

d. From the intersection of *ob* and *bc*, draw *oc* parallel to *OC* until it intersects *cd*; from the intersection, draw *od* parallel to *OD* to intersect *de*, then draw *oe* parallel to *OE* to intersect *oa* at *K*. The line of action *ae* passes through *K*, 5.3 feet to the right of *ab*.

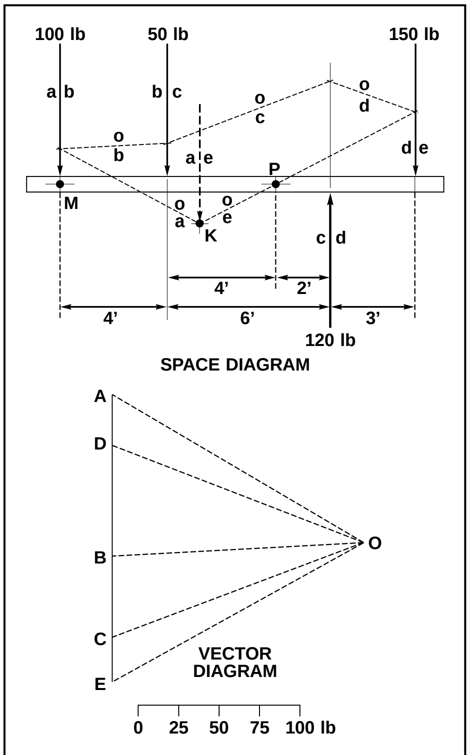

Figure D-9. Funicular Diagram.

To solve the problem algebraically, a sign convention is adopted (upward forces positive, downward forces negative) and the magnitude and sense of the resultant is given by the sum of the forces ($R = \Sigma F$). The line of action of the resultant is determined by summing moments. All moment arms are measured from the same point, selected to simplify calculations, and the sum of the moments divided by the resultant force gives the resultant moment arm, locating the line of action of the resultant, where *resultant arm* = $\Sigma M/R$.

If the summation of forces is zero, but the sum of moments is not zero, the resultant is a couple.

D-3.2.5 Noncoplanar Nonconcurrent Forces. The resultant of a system of noncoplanar parallel forces may be either a single force or a couple. A set of three rectangular axes is established, with the *Z*-axis parallel to the lines of action of the forces. The intersection of the lines of action of the forces with the *x-y* plane is indicated by *x, y* coordinates. Moment arms about the *X* and *Y* axes are readily determined from the coordinates for each force. As with coplanar force systems, the magnitude and sense of the resultant are given by the summation of forces ($R = \Sigma F$). The coordinates of the line of action of the resultant are found by dividing the sums of the moments about the *X* and *Y* axes by the resultant:

$$x_R = \frac{\Sigma M_y}{R} = \frac{\Sigma(F_n x_n)}{\Sigma F}, \qquad y_R = \frac{\Sigma M_x}{R} = \frac{\Sigma(F_n y_n)}{\Sigma F}$$

If $\Sigma F = 0$, but $\Sigma M_x \neq 0$ or $\Sigma M_y \neq 0$, the resultant is a couple in a plane parallel to the *Z*-axis. The moment of the couple, and its orientation with the *x-z* plane, is determined by omitting one of the forces from the force and moment summations, as shown in the following example.

EXAMPLE D-3

The lines of action of the four forces shown in the vector diagram in Figure D-10 are perpendicular to the plane of the paper. Positive and negative senses are as shown. Summing forces and moments:

	Force lbs	x ft	M_y ft-lb	y ft	M_x ft-lb
	+ 20	- 1	- 20	+ 2	+ 40
	+ 40	- 1	- 40	- 1	- 40
	- 35	+ 1	- 35	+ 1	- 35
	- 25	+ 2	- 50	+ 2	- 50
Sums	0		- 145		- 85

$\Sigma F = 0$, and ΣM_x, $\Sigma M_y \neq 0$; the resultant is a couple. If the last force is omitted, R_o = +25 lb, ΣM_y = -95 ft-lb, and ΣM_x = -35 ft-lb. The coordinates of the line of action of R_o are:

$$x_o = \frac{\Sigma M_y}{R} = \frac{-95}{+25} = -3.8 \text{ ft}, \qquad y_o = \frac{\Sigma M_x}{R} = \frac{-35}{+25} = -1.4 \text{ ft}$$

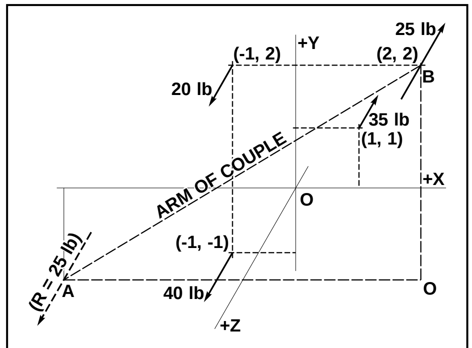

Figure D-10. Noncoplaner, Nonconcurrent Forces.

The arm of the couple is the distance *AB*, from the resultant force to the most distant opposite force. *AB* is determined to be 6.72 feet from triangle *ABC*. The moment of the resultant couple is 25(6.72) = 168 ft-lb. The angle *BAC* is the angle between the plane of the couple and *x-z* plane. Angle *BAC* = arctan 3.4/5.8 = 30.38 degrees. The sense of the couple is seen to be counter-clockwise when viewed from the positive end of the *Y*-axis.

The resultant of a system of noncoplanar, nonconcurrent, nonparallel forces is generally a single force and a couple not coplanar with the force. The magnitude, sense, and angular direction of the force is the same as if the forces were concurrent:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

$$\phi_x = \arccos \frac{\Sigma F_x}{R} \quad \phi_y = \arccos \frac{\Sigma F_y}{R} \quad \phi_z = \arccos \frac{\Sigma F_z}{R}$$

R acts through the selected reference origin. The couple is determined by summing moments about the coordinate system axes. The moment sums represent three couples which are axial components of the resultant couple. If ΣM_x is taken as a vector along the *X*-axis, ΣM_y as a vector along the *Y*-axis, and ΣM_z along the *Z*-axis, then the moment of the resultant couple and the direction angles of its vector are given by:

$$C = \sqrt{(\Sigma M_x)^2 + (\Sigma M_y)^2 + (\Sigma M_z)^2}$$

$$\theta_x = \arccos \frac{\Sigma M_x}{C} \quad \theta_y = \arccos \frac{\Sigma M_y}{C} \quad \theta_z = \arccos \frac{\Sigma M_z}{C}$$

R and C can be compounded into two nonintersecting forces.

D-3.3 Conditions of Equilibrium. A body is in equilibrium with respect to some reference system if it does not move with respect to the reference (*static* equilibrium), or moves with constant velocity (*dynamic* equilibrium). For an object to be in equilibrium, the resultant of all external forces and moments must be zero.

Table D-2. Conditions of Equilibrium.

System	Algebraic Conditions	Graphical Conditions
Coplanar		
Colinear	$\Sigma F = 0$.	Force polygon closes.
Concurrent at point O	$\Sigma F_x = 0$, $\Sigma F_y = 0$, if the angle between x and y is not 180 degrees; or $\Sigma F_x = 0$, $\Sigma M_a = 0$, if the x direction is not perpendicular to *Oa*; or $\Sigma M_a = 0$, $\Sigma M_b = 0$, if *aOb* is not a straight line.	Force polygon closes.
Parallel	$\Sigma F = 0$, $\Sigma M = 0$; or $\Sigma M_a = 0$, $\Sigma M_b = 0$, if line *ab* is not parallel to the forces.	Force polygon closes, funicular polygon closes (first and last strings coincide).
Nonparallel, nonconcurrent	$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$; or $\Sigma F_x = 0$, $\Sigma M_a = 0$, $\Sigma M_b = 0$, if x is not perpendicular to *ab*; or $\Sigma M_a = 0$, $\Sigma M_b = 0$, $\Sigma M_c = 0$, if *abc* is not a straight line.	Force and funicular polygons close.
Noncoplanar		
Concurrent at point O	$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$; or ΣF_s in every direction and ΣM_n about every axis = 0.	Force polygon closes. Polygon is warped, so both plan and elevation views must close.
Parallel	$\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$, forces parallel to *Z*-axis.	Not used.
Nonparallel, nonconcurrent	$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$. ΣM about every axis = 0.	The projection of the system on any plane is in equilibrium.

Depending on the kind of force system involved, different tests or conditions are applied to *prove* that the system is in equilibrium. In most cases, a body is determined to be in equilibrium by inspection, and the applicable conditions of equilibrium are used to develop relationships that can be solved for unknown forces, moments, distances, or angles. Conditions of equilibrium for various force systems are shown in Table D-2.

If three forces are in equilibrium, they must be coplanar, and either concurrent or parallel. If concurrent, each force is proportional to the sine of the angle between the other two forces. If parallel, each force is proportional to the distance between the other two.

If a force system is in equilibrium, the resultant of any part must balance the resultant of the other part. This fact is the basis for the construction of *free-body diagrams*. A free-body diagram shows an object in equilibrium, with all external forces, moments, and support reactions. With the object in equilibrium, the resultant of all forces and moments on the free body is zero. If any part of the object is removed and replaced by the forces and moments exerted by the "cut" surface, a free-body diagram of the remaining structure is obtained, and the conditions of equilibrium are satisfied by the new free body. By dividing an object into a sufficient number of free bodies, internal forces and moments can be determined at all points of interest, provided the conditions of equilibrium are sufficient to give a static solution.

D-3.4 Centroids and Centers of Gravity. The *centroid* of a system of parallel forces with fixed application points is the point through which their resultant always passes, no matter how the lines of action of the forces may be rotated, so long as they remain parallel. For plane surfaces, the centroid corresponds to the center of area, so long as the forces are not affected by geometry; for volumes the centroid is the center of volume. Determination of centers of areas and volumes by numerical integration is discussed in Paragraph 1-4. Relationships for locating the centroids of various plane shapes are given in Appendix C of the *U.S. Navy Ship Salvage Manual, Volume 1,* S0300-A6-MAN-010.

The force of gravity acting on individual particles of a body constitutes a system of very nearly parallel forces; the centroid of these forces is the *center of gravity* of the body. Calculation of center of gravity is discussed in Paragraph 1-3.7.

D-3.5 Moment of Inertia. Moment of inertia is a measure of the resistance of a solid or plane area to rotation about axes in the plane of the area considered, and is always positive. The moment of inertia of a solid body, sometimes called the *mass moment of inertia* (I_m) with respect to a given axis is the sum of the products of the masses of each elemental mass of which the body is composed and the square of the distance of each element from the axis. If dm is an elemental mass, and y its distance from a reference axis, the moment of inertia of the body about the axis is $I_{\mathrm{m}} = \int y^2\, dm$. I_m is measured in units of mass and length squared, such as slug-ft^2. Moment of inertia can also be expressed as $I_m = k^2 m$, where m is the mass of the body and k is the radius of gyration or radius of inertia. The radius of gyration is the distance from the axis to a point at which the mass of the body could be concentrated without changing the moment of inertia. k is measured in units of length, and lies between the greatest and lowest values of y. If a body is composed of a number of parts, its moment of inertia about an axis is equal to the sum of the moments of inertia of the individual parts about the same axis.

The moment of inertia (I) of a plane surface with respect to a given axis is the sum of the products of the incremental areas of which the surface is composed and the square of the distance of the incremental areas from the axis. If dA is an incremental area, and y its distance from a reference axis, the moment of inertia of the surface about the axis is $I = \int y^2\, dA = k^2 A$, where A is the total area, and k is the radius of gyration. The quantity $\int y^2\, dA$ is more properly referred to as the *second moment of area*, as it has units of length to the fourth power and is not truly a measure of *inertia*. For homogeneous solids of uniform thickness, the mass moment of inertia is equal to the moment of inertia of the face of the solid, multiplied by the mass per unit volume and thickness of the solid. Moments of inertia for structural shapes are tabulated in Appendix E. Calculation of moment of inertia for ship sections is discussed in Paragraph 1-11. Relationships for moments of inertia of various plane shapes are given in Appendix C of the *U.S. Navy Ship Salvage Manual, Volume 1* (S0300-A6-MAN-010).

D-3.5.1 Parallel Axis Theorem. The moment of inertia of an area or mass is equal to the moment of inertia about a parallel axis through the center of gravity, plus the product of the area or mass, and the square of the distance between the two axes:

$$I_{AA} = I_o + Ad^2 , \qquad I_{m,AA} = I_{m,o} + md^2$$

where:

I_{AA} = moment of inertia (second moment) of area about some axis AA
I_o = moment of inertia (second moment) of area about an axis parallel to AA through the center of area (centroid)
A = total area
d = perpendicular distance from the center of area or gravity to axis AA
$I_{m,AA}$ = moment of inertia of mass about some axis AA
$I_{m,o}$ = moment of inertia of mass about an axis parallel to AA through the center of gravity (not necessarily the center of volume)

D-3.5.2 Polar Moment of Inertia. The polar moment of inertia is taken about an axis perpendicular to the plane of the area and is a measure of the area's resistance to twisting in its own plane of the area. The polar moment of inertia (I_p or J) is equal to the sum of the moments of inertia about any two mutually perpendicular axes in the plane of the area that pass through the center of area:

$$J = I_p = I_{XX} + I_{YY}$$

D-3.5.3 Product of Inertia. The product of inertia (I_{xy}), sometimes called the *cross moment of inertia*, is equal to $\iint xy\,dy\,dx$, where x and y are the coordinates of incremental areas. I_{xy} may be positive or negative, depending on the location of the area with respect to the reference axes XX and YY.

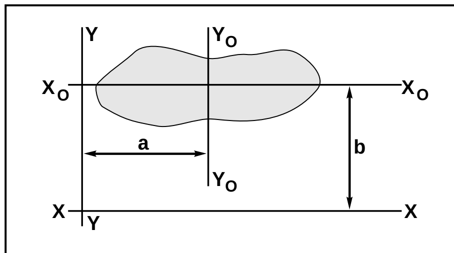

Figure D-11. Product of Inertia and Parallel Axes.

If $I_{XY,O}$ is the product of inertia of area A about the mutually perpendicular axes X_oX_o, Y_oY_o through the center of area as shown in Figure D-13, and axes XX, YY are parallel to X_oX_o, Y_oY_o, then:

$$I_{XY} = I_{XY_0} + abA$$

where I_{xy} is the product of inertia of area A about axes XX and YY and a and b are shown in Figure D-11.

D-3.5.4 Moments of Inertia About Inclined Axes. If I_x and I_y are moments of inertia about a set of mutually perpendicular axes XX and YY, and $X'X'$, $Y'Y'$ are a set of mutually perpendicular axes inclined at some angle θ to XX and YY, then:

$$I_y' = I_y \cos^2\theta + I_x \sin^2\theta + I_{xy} \sin 2\theta$$

$$I_x' = I_x \cos^2\theta + I_x \sin^2\theta + I_{xy} \sin 2\theta$$

$$I_{xy}' = \frac{I_x - I_y}{2} \sin 2\theta + I_{xy} \cos 2\theta$$

where:

I_y' = moment of inertia about axis $Y'Y'$
I_x' = moment of inertia about axis $X'X'$
I_{xy}' = product of inertia about axes $X'X'$, $Y'Y'$

D-3.5.5 Principal Moments of Inertia. For any plane area, there is a set of mutually perpendicular axes such that the moment of inertia is maximum about one axis and minimum about the other. These axes are the *principal axes of inertia*, and the corresponding moments of inertia are the *principal moments of inertia*. The product of inertia about the principal axes of inertia is zero. Axes of symmetry are always principal axes of inertia.

D-4 PHYSICAL AND MECHANICAL PROPERTIES OF MATTER

D-4.1 Density. *Density* (ρ) is the mass of a unit volume. Typical units are slugs per cubic foot, kilograms per cubic meter, grams per cubic centimeter, and pounds-mass per cubic foot. Many fluid flow calculations are based on density measured in pounds-mass per cubic foot.

Weight density, or *specific weight,* is given by:

$$\gamma = \frac{\rho g}{g_c}$$

In a standard gravitational field (g = 32.174 ft/sec^2), weight density in pounds-force and mass density in pounds-mass are numerically equal.

Density of a liquid or solid is usually given, obtained from a table, or easily determined by weighing a sample of known volume. The density of a gas can be found from a modification of the ideal gas law:

$$\rho_{gas} = \frac{p}{RT}$$

where:

ρ_{gas} = gas density, lbm/ft^3
p = pressure, lbf/ft^2
R = specific gas constant, ft-lbf/lbm-°R
T = absolute temperature, °R

Tables E-19 and E-20 give densities of common solids and liquids. More extensive tables can be found in general engineering and technical handbooks (see Bibliography).

D-4.1.1 Specific Volume. *Specific volume* (δ) is the volume occupied by a unit mass or weight, and is the reciprocal of density:

$$\delta = \frac{1}{\rho} \approx \frac{1}{\gamma}$$

D-4.1.2 Specific Gravity. *Specific gravity* (γ_g), sometimes called *relative density,* is the ratio of a fluid's density to a specified reference density. For liquids and solids, the normal reference is the density of pure water. There is some confusion about this reference since the density of water varies with temperature, and various reference temperatures have been used (e.g., 39, 60, 70 degrees Fahrenheit, etc.).

Strictly speaking, specific gravity cannot be given without specifying the reference temperature at which the water's density was evaluated. However, the reference temperature is often omitted since water's density is fairly constant over the normal ambient temperature range. To three significant digits, the reference density is 62.4 lbm/ft^3, and:

$$\gamma_g = \frac{\rho}{62.4}$$

Specific gravities of petroleum products and aqueous acid solutions are routinely expressed in "degrees" corresponding to hydrometer readings. The principal hydrometer scale in current use is the API (American Petroleum Institute) scale, although the Baumé scale was used widely in the past. API gravities are converted to specific gravity (ratio) by:

$$\gamma_g = \frac{141.5}{131.5 + °\text{API}}$$

Baumé hydrometer readings are converted to specific gravity by:

$$\gamma_g = \frac{140.0}{130.0 + °\text{Baumé}} \qquad \text{for liquids less dense than water}$$

$$\gamma_g = \frac{145.0}{145.0 - °\text{Baumé}} \qquad \text{for liquids denser than water}$$

Appendix B of the *U.S. Navy Ship Salvage Manual, Volume 5,* (S0300-A6-MAN-050) includes tables converting API gravities to specific gravity and density and temperature corrections for observed API gravities.

The reference density for the specific gravity, or *vapor density*, of a gas is normally the density of air at specified conditions of pressure and temperature. The most commonly used reference is air at standard temperature and pressure (STP), i.e. 70 degrees Fahrenheit and atmospheric pressure. The density of air at STP is approximately 0.075 lbm/ft^3, and:

$$\gamma_{gas} \approx \frac{\rho}{0.075}$$

If the gas and air densities are evaluated at the same temperature and pressure, the specific gravity is the inverse ratio of specific gas constants:

$$\gamma_{gas} \approx \frac{R_{air}}{R_{gas}} = \frac{53.3}{R_{gas}}$$

D-4.2 Viscosity. The *viscosity* of a fluid is a measure of its resistance to flow. Viscosity is illustrated by a model consisting of two plates that are separated by a viscous fluid layer with thickness (y). The bottom plate is fixed; the top plate is kept in motion at a constant velocity (v) by a constant force (F).

Experiments with Newtonian fluids have shown that the force required to maintain the velocity is proportional to the velocity and inversely proportional to the separation of the plates. That is,

$$\frac{F}{A} \propto \frac{d\mathrm{v}}{dy}$$

The constant of proportionality, designated by the symbol μ, is known as the *absolute* or *dynamic* viscosity. Noting that the quantity *F/A* is the fluid shear stress:

$$\tau = \mu \frac{d\mathrm{v}}{dy}$$

Kinematic viscosity is defined as:

$$\nu = \frac{\mu g_c}{\rho}$$

Viscosity is measured in a variety of units. Table D-3 lists the most commonly used units in the English and SI systems.

Conversions between the two types of viscosities and between the English and various metric systems are given in Table D-4.

Table D-3. Viscosity Units.

System of Units	Absolute Viscosity μ	Kinematic Viscosity ν
English	lbf-sec/ft^2 slug/ft-sec	ft^2/sec
Metric - CGS	dyne-sec/cm^2 (poise)	cm^2/sec (stoke)
Metric - SI	Pascal-sec N-sec/m^2	m^2/sec

Table D-4. Viscosity Conversions.

To Obtain	Multiply	By	and Divide by
ft^2/sec	lbf-sec/ft^2	32.174	density
ft^2/sec	stokes	1.076×10^{-3}	1
lbf-sec/ft^2	ft^2/sec	density	32.2
lbf-sec/ft^2	poise	1	478.8
m^2/s	centistokes	1×10^{-6}	1
m^2/s	stokes	1×10^{-6}	1
m^2/s	ft^2/sec	9.29×10^{-2}	1
pascal-sec	centipoise	1×10^{-3}	1
pascal-sec	lbm/ft-sec	1.488	1
pascal-sec	lbf-sec/ft^2	47.88	1
pascal-sec	poise	.1	1
pascal-sec	slug/ft-sec	47.88	1
poise	lbf-sec/ft^2	478.8	1
poise	stokes	specific gravity	1
reyns	lbf-sec/ft^2	1	144
stokes	ft^2/sec	929	1
stokes	poise	1	specific gravity

Kinematic viscosity is measured indirectly by a viscometer, a container which allows the fluid to leak out through a small orifice of precise dimensions. The more viscous the fluid, the more time will be required to leak out a given quantity. Viscosity measured in this manner has the units of seconds. The standard viscosimeters in the United States are the Saybolt Universal viscosimeter for ordinary liquids, and the Saybolt Furol viscosimeter for viscous liquids. The time required for a gravity flow of 60 cubic centimeters through the orifice is called Saybolt Seconds Universal (SSU) or Saybolt Seconds Furol (SSF). Saybolt Universal viscosimeters are calibrated so that the viscosity of pure water is 30 SSU. Approximate conversion of SSU and SSF to stokes may be made by:

$$\text{stokes} = 0.00226\,\text{SSU} - \frac{1.95}{\text{SSU}} \qquad (32 < \text{SSU} < 100)$$

$$= 0.00220\,\text{SSU} - \frac{1.35}{\text{SSU}} \qquad (\text{SSU} > 100)$$

$$\text{stokes} = 0.0224\,\text{SSF} - \frac{1.84}{\text{SSF}} \qquad (25 < \text{SSF} < 40)$$

$$= 0.0216\,\text{SSF} - \frac{0.60}{\text{SSF}} \qquad (\text{SSF} > 40)$$

In liquids, molecular cohesion is the dominating cause of viscosity. As the temperature of a liquid increases, these cohesive forces decrease and absolute viscosity decreases.

In gases, the dominating cause of viscosity is random collisions between gas molecules. This molecular agitation increases with temperature, causing the viscosity of gases to also increase with temperature.

The absolute viscosity of both gases and liquids is independent of pressure. Kinematic viscosity depends on both temperature and pressure because these variables affect density.

D-4.3 Vapor Pressure. Molecular activity in a liquid tends to free some surface molecules that enter the atmosphere as vapor. This tendency toward vaporization increases with temperature. *Vapor pressure* is the partial pressure exerted at the surface by the free molecules. Boiling occurs when liquid vapor pressure exceeds the local ambient pressure.

D-4.4 Surface Tension. The skin which seems to form on the free surface of a fluid is due to the intermolecular cohesive and adhesive forces known as surface tension. Surface tension is the amount of work required to form a new unit of surface area. The units are ft-lbf/ft^2 or lbf/ft.

Surface tension can be measured as the tension between two points on the surface separated by a foot. It decreases as temperature increases and depends on the gas contacting the free surface. Surface tension values usually are quoted for air contact. Typical values are given in Table D-5.

Table D-5. Typical Surface Tensions (68 °F, Air Contact).

Fluid	*T*
Ethyl alcohol	.001527 lbf/ft
Turpentine	.001857
Water	.004985
Mercury	.03562
N-octane	.00144
Acetone	.00192
Benzene	.00192
Carbon tetrachloride	.00180

The relationship between surface tension and the pressure in a bubble surrounded by gas is given by:

$$T = \frac{1}{4} r (p_{inside} - p_{outside})$$

where r is the radius of the bubble. The surface tension in a full spherical droplet or in a bubble in a liquid is given by:

$$T = \frac{1}{2} r (p_{inside} - p_{outside})$$

Surface tension is the cause of *capillarity*, which occurs whenever a liquid comes into contact with a vertical solid surface. In water, adhesive forces dominate. They cause water to attach itself readily to a vertical surface and climb the wall. In a thin-bore tube, water will rise above the general level as it tries to wet the interior surface.

D-4.5 Compressibility. Compressibility is the percentage change in a unit volume per unit change in pressure:

$$C = \frac{\frac{\Delta V}{V}}{\Delta p}$$

Liquids are usually considered incompressible, but all fluids are somewhat compressible. The bulk modulus is the reciprocal of the compressibility:

$$E = \frac{1}{C}$$

The bulk modulus of an ideal gas is given by:

$$E = kp$$

where p is absolute pressure and k is the ratio of specific heats; k is 1.4 for air.

D-5 FLUID MECHANICS

Fluids are generally divided into two categories: *ideal* and *real*. Ideal fluids have zero viscosity and shearing forces, are incompressible, and have uniform velocity distributions when flowing.

Real fluids are divided into Newtonian and non-Newtonian fluids. Both Newtonian and non-Newtonian fluids have finite viscosities and nonuniform velocity distributions when flowing. Viscosities of Newtonian fluids are independent of the rate of change of shear stress, while viscosities of non-Newtonian fluids vary with the rate of change of shear stress. Newtonian fluids are typified by gases, thin liquids, and most fluids having simple chemical formulas. Non-Newtonian fluids are typified by gels, emulsions, and suspensions.

Most fluid problems assume Newtonian fluid characteristics.

D-5.1 Fluid Statics. Pressures are measured as standard or absolute. *Absolute pressures* are measured from a reference datum of zero absolute pressure; there are no negative pressures. *Gage pressures* are measured from standard atmospheric pressure (approximately 14.7 psia). Negative gage pressures (below atmospheric pressure) are called vacuum. Maximum vacuum is therefore -14.7 psig.

D-5.1.1 Manometers. Manometers measure pressure differentials. Figure D-12 shows a simple U-tube manometer whose ends are connected to two pressure vessels. If one end is open to the atmosphere, the manometer measures the difference between pressure at the other end and atmospheric pressure, i.e., gage pressure. Since the pressure at point *B* is the same as at point *C*, the height *(h)* of the fluid column is related to the pressure differential (Δp):

$$\Delta p = p_2 - p_1 = \gamma_m h$$

where γ_m is the weight density of the manometer fluid. This relationship assumes that the manometer is small and that only low-density gases fill the tubes above the measuring fluid. If a high-density fluid (such as water) is present above the measuring fluid, or if the gas columns h_1 or h_2 are very long, a correction is required:

$$\Delta p = \gamma_m h + \gamma_1 h_1 - \gamma_2 h_2$$

where γ_1 is the density of the fluid above the high end of the measuring fluid and γ_2 is the density of the fluid above the low end of the manometer fluid; h_1 and h_2 are the heights of the fluid columns above the measuring fluid, as shown in Figure D-13. Corrections for capillarity are seldom needed, since manometer tubes generally are large enough to preclude capillary action.

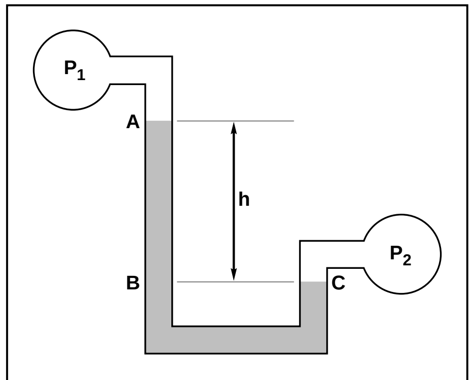

Figure D-12. Simple Manometer.

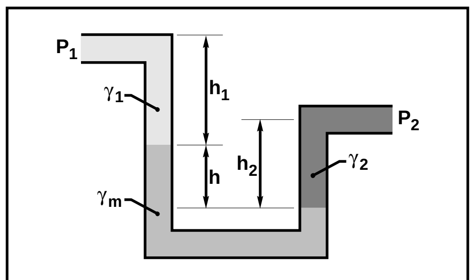

Figure D-13. Manometer with Fluids Over Measuring Fluid.

D-5.1.2 Hydrostatic Pressure From Incompressible Fluids. Hydrostatic pressure is the pressure which a fluid exerts on an object or container walls. Its line of action is normal to the exposed surface, regardless of the object's orientation or shape. It varies linearly with depth and is a function of depth and density only. Pressure acting on an incremental area creates an incremental pressure force; the resultant of all the incremental forces, or *net hydrostatic force,* is a function of pressure and area distribution and acts through the center of pressure.

Pressure on a horizontal surface uniform and constitutes a system of parallel forces; the center of pressure is the centroid of the plane surface. The gage pressure and total vertical force are given by:

$$p = \gamma h, \qquad F = pA$$

where:

p	=	hydrostatic pressure, lb/ft^2
γ	=	fluid (weight) density, lb/ft^3
h	=	depth of fluid of the surface, ft
F	=	hydrostatic force, lbf
A	=	area of the plane surface, ft^2

For a rectangular plate immersed in a fluid body, either vertically or inclined at some angle θ, as shown in Figure D-14, pressure varies linearly with depth. The pressures at the top and bottom of the plate are:

$$p_1 = \gamma h_1 = \gamma s_1 \sin\theta$$

$$p_2 = \gamma h_2 = \gamma s_2 \sin\theta$$

where subscripts 1 and 2 denote the top and bottom of the plate, respectively, and s is the distance from the intersection of the liquid surface and the extension of the plate surface to the point in question, measured parallel to the plate surface. The average pressure occurs at the average depth $(1/2)(h_1 + h_2)\sin\theta$ and is equal to:

$$p_{avg} = \frac{1}{2}\gamma(h_1 + h_2) = \frac{1}{2}\gamma(s_1 + s_2)\sin\theta$$

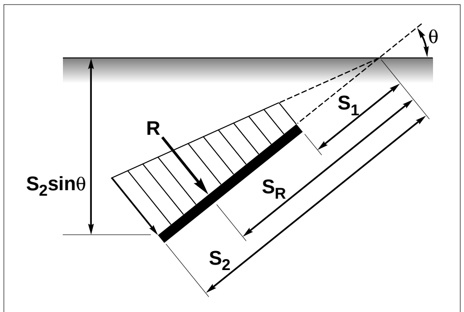

Figure D-14. Immersed Rectangular Plate.

The total resultant force on the inclined plane is the product of the average pressure and total area, $F = p_{avg}A$. The center of pressure is not located at the average depth but at the depth of the center of the triangular or trapezoidal pressure distribution:

$$s_R = \frac{2}{3}\left(s_1 + s_2 - \frac{s_1 s_2}{s_1 + s_2}\right)$$

$$h_R = s_R \sin\theta$$

For a nonrectangular plane surface, the average pressure depends on the location of the centroid of the surface (s_c):

$$p_{avg} = \gamma s_c \sin\theta, \qquad F = p_{avg}A$$

The line of action of the resultant (hydrostatic force) is normal to the plane surface, at depth h_R:

$$s_R = s_c + \frac{I_c}{As_c}, \qquad h_R = s_R \sin\theta$$

where I_c is the moment of inertia about an axis parallel to the surface through the area's centroid.

D-5.1.3 Hydrostatic Pressure From Compressible Fluids. The expression $p = \gamma h$ is a special case of the more general Fundamental Equation of Fluid Statics:

$$\int_1^2 \frac{dp}{p} = -(h_2 - h_1)$$

As previously defined, h is depth within the fluid, and it is assumed that h_2 is greater than h_1. The minus sign indicates that pressure decreases when height increases. If the fluid is a compressible layer of perfect gas, and if compression is assumed to be isothermal, the Equation of Fluid Statics becomes:

$$h_2 - h_1 = RT \ln\left(\frac{p_1}{p_2}\right)$$

The pressure at height h_2 in a layer of isothermally compressed gas is:

$$p_2 = p_1\left(e^{\frac{h_1 - h_2}{RT}}\right)$$

EXAMPLE D-4

The pressure at sea level is 14.7 psia. Assume 70 °F isothermal compression, and calculate the pressure at 5,000 feet altitude.

$R = 53.3$ ft-lbf/lbm - °R for air. $T = (70 + 460) = 530°R$.

$$p_{5000\text{ft}} = 14.7\left(e^{\frac{0 - 5000}{(53.3)(530)}}\right) = 12.32 \text{ psia}$$

D-5.1.4 Fluid Masses Under Acceleration. The equations presented to this point have assumed that the fluid is subjected only to gravitational acceleration. When a fluid is subjected to other accelerations, additional forces, which change hydrostatic pressures, are imposed.

If the fluid is subjected to constant accelerations in the vertical and/or horizontal directions, fluid behavior is given by:

$$p_h = \gamma h \left(1 + \frac{a_y}{g}\right), \quad \theta = \arctan\left(\frac{a_x}{a_y + g}\right)$$

where a_y is the vertical acceleration (negative if the acceleration is downward) and a_x the horizontal acceleration. θ is the angle between the liquid surface and the horizontal, as shown in Figure D-15. A plane of equal pressure also is inclined in a fluid mass under horizontal acceleration.

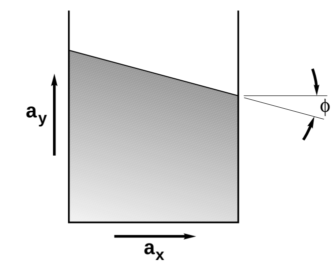

Figure D-15. Fluid Mass Under Constant Linear Acceleration.

D-5.2 Head. Pressure is measured in units of force per unit area (pounds per square inch, pounds per square foot, newtons per square meter, etc.). Pressure is converted to the new variable *head* by dividing by the fluid density. Since density itself possesses dimensional units, the units of head are not the same as the units of pressure:

$$(h, \text{ft}) = \frac{(p, \text{lbf/ft}^2)}{(\gamma, \text{lbf/ft}^3)} \approx \frac{p, \text{ lbf/ft}^2}{\rho \text{lbm/ft}^3}$$

As long as the fluid density and local gravitational acceleration remain constant, there is complete numerical interchangeability between pressure and head. Head is used as a measure of specific energy:

$$(h, \text{ft}) = \frac{(E, \text{ft-lbf})}{(\text{mass}, \text{lbm})}$$

A certain amount of care in the use of these equations is required, because lbf is being canceled by lbm. The actual cancellation is:

$$h \text{ in ft} = \frac{\left(g_c, \frac{\text{lbm-ft}}{\text{lbf-sec}^2}\right)\left(p, \frac{\text{lbf}}{\text{ft}^2}\right)}{\left(g, \frac{\text{ft}}{\text{sec}^2}\right)\left(\rho, \frac{\text{lbm}}{\text{ft}^3}\right)}$$

As g_c always equals 32.174, the correct numerical value for head will be returned as long as the local gravitational acceleration is 32.174 ft/sec^2.

D-5.3 Reynolds Number. The Reynolds number is a dimensionless ratio of the inertial flow forces to the viscous forces within the fluid:

$$R_e = \frac{D_e V \rho}{\mu g_c} = \frac{D_e V}{\nu}$$

where:

D_e	=	equivalent flow diameter
V	=	flow velocity
ρ	=	fluid density
μ	=	absolute viscosity of the fluid
ν	=	kinematic viscosity of the fluid

The Reynolds number can be calculated from the unit mass flow rate (G):

$$R_e = \frac{D_e G}{\mu g_c}$$

where:

D_e	=	equivalent flow diameter, ft
V	=	flow velocity, ft/sec
G	=	mass flow rate per unit area, lbm/sec-ft^2
μ	=	absolute viscosity of the fluid, lbf-sec/ft^2
g_c	=	gravitational constant = 32.174 lbm-ft/lbf-sec^2

The Reynolds number is an important indicator in many types of problems. In addition to being used quantitatively in many equations, the Reynolds number also is used to determine whether fluid flow is *laminar* or *turbulent*. A Reynolds number of 2,000 or less indicates laminar flow. Fluid particles in laminar flow move in straight paths parallel to the flow direction. Viscous effects are dominant, resulting in a parabolic velocity distribution with a maximum velocity along the fluid flow centerline. If the Reynolds number is greater than 4,000, flow is turbulent. Turbulent flow is characterized by random movement of fluid particles. For Reynolds numbers between 2,000 and 4,000, the flow regime is in transition from laminar to turbulent flow.

D-5.4 Equivalent Diameter. For a circular flow channel, the equivalent diameter (D_e) in the expressions for Reynolds number is the inside diameter. Equivalent diameters for other shaped channels are given in Table D-6.

Table D-6. Equivalent Diameters.

Conduit Cross Section	D_e	Conduit Cross Section	D_e
Flowing Full		**Flowing Partially Full**	
Annulus	$D_o - D_i$	Half-filled circle	D
Square	L	Rectangle (h deep, L wide)	$\frac{4hL}{L + 2h}$
Rectangle	$\frac{2L_1L_2}{L_1 + L_2}$	Wide, shallow stream (h deep)	$4h$
		Triangle (h deep, L broad, s side)	$\frac{hL}{s}$
		Trapezoid (h deep, a wide at top, b wide at bottom, s side)	$\frac{2h(a + b)}{b + 2s}$

D-5.5 Hydraulic Radius. The hydraulic radius (r_h) of a flow channel is the area in flow divided by the wetted perimeter, exclusive of the free liquid surface. Equivalent diameter can be found from the hydraulic radius:

$$D_e = 4r_h$$

D-6 STRENGTH OF MATERIALS

External forces acting on a body are resisted by reactions within the body, termed *stresses*. The maximum stress that can be sustained by a material is the measure of its strength, and is determined by the elastic and cohesive properties of the material.

D-6.1 Stress. Stress is defined as force (*F*) per unit area (*A*) and thus has the same units as pressure. Conditions causing the three fundamental types of stress are illustrated in Figure D-16. *Normal* or *axial* stresses (tensile and compressive) result from forces acting at right angles to the cross section, and are indicated by the symbol σ, *s*, or *f*. The average normal stress created by a force (*F*) acting on a cross section of area (*A*) is:

$$\sigma = \frac{F}{A}$$

In most calculations, tensile stress taken as positive and compressive stress as negative. Shear stresses result from forces acting parallel to the cross section, and are indicated by the symbol τ, s_t, or *q*:

$$\tau = \frac{F}{A_s}$$

where τ is the average shear stress in area A_s that is being sheared by force *F*. Bearing stress is actually a pressure, as it is the intensity of force between a body and its support. Bearing stress is indicated by the symbol σ_b or s_b, and, like normal and shear stress, is defined as a ratio of force to area.

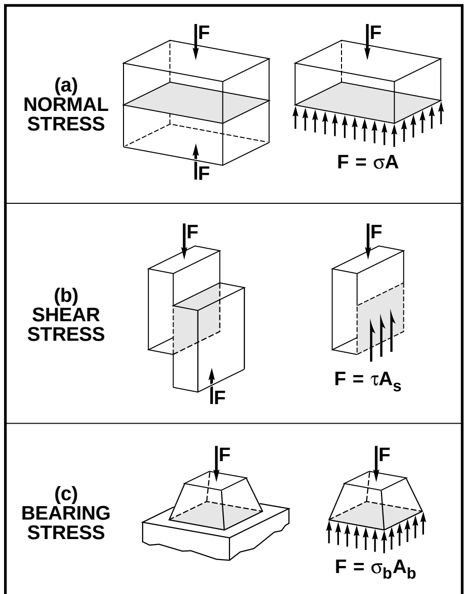

Figure D-16. Stresses.

D-6.2 Strain. Strain (ε) is deformation expressed as a pure number or ratio. For a member in tension or compression, it is expressed as the change in length divided by original length. *True* strain (δ) is the logarithm of the ratio of the length at the moment of observation to the original length. True strain (δ) does not differ much from ε until above 20 percent. Elongation is accompanied by a reduction in cross-sectional area. *Poisson's ratio* (μ) is the ratio of strain measured at right angles to the applied stress to strain measured parallel to the applied stress—essentially a statement of constancy of volume during deformation. For elastic strain, μ ranges from 0.283 to 0.292 for most structural steels, and from 0.330 to 0.334 for most aluminum alloys. For plastic strain, μ is approximately 0.5.

D-6.3 Stress-Strain Relationships. Stress-strain relationships differ slightly for tensile and compressive stress.

D-6.3.1 Tensile Stress-Strain Diagram. The commonly used engineering tensile stress-strain curve is obtained by statically loading a standard specimen; that is, by loading the specimen slowly enough that all parts of the specimen remain in static equilibrium. Figure D-17(a) shows stress-strain curves for several metals.

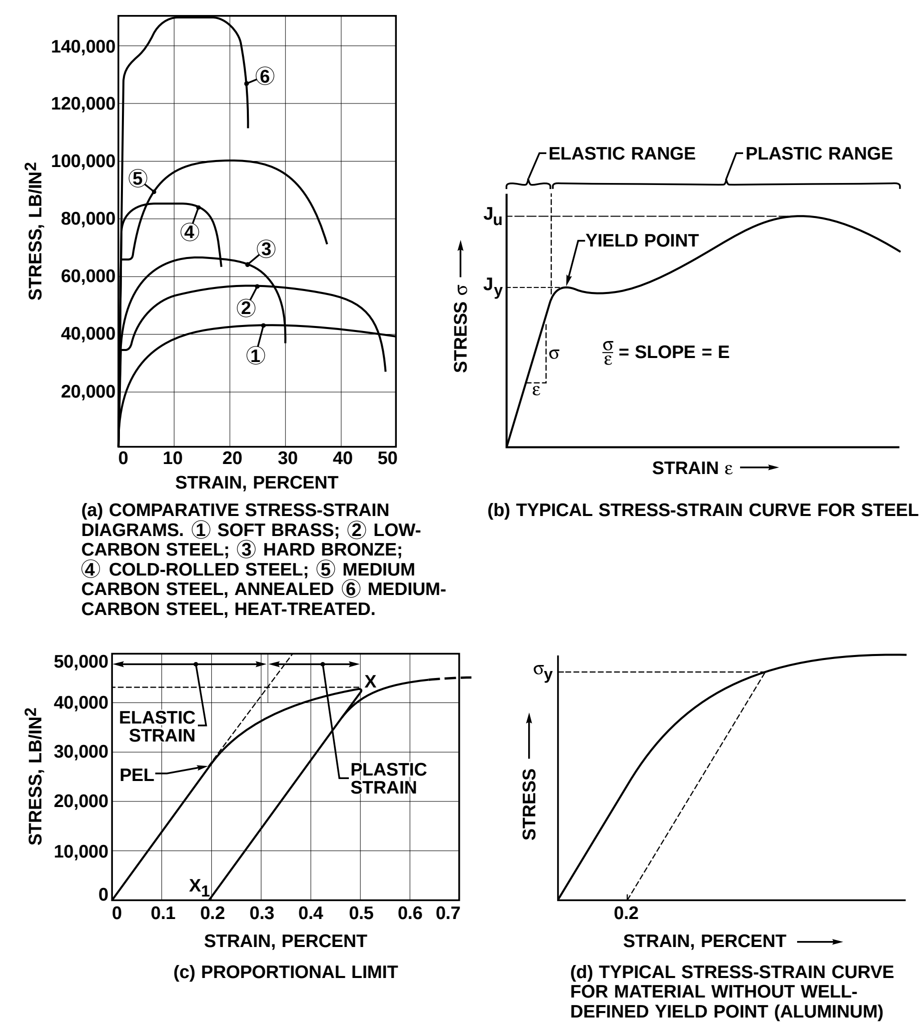

Figure D-17. Stress-Strain Relationships.

Stress-strain curves for most engineering materials have an initial linear *elastic* region, as shown in Figure D-17, where deformation is reversible and time-independent. The slope of this portion of the curve, stress divided by unit elongation, is the *modulus of elasticity*, or *Young's modulus*. In the elastic region, strain is proportional to stress, and the material is said to follow *Hooke's Law*. The proportional limit is the point where the curve begins to deviate from a straight line, i.e., the point where strain ceases to be proportional to stress. The *elastic limit* is the maximum stress that a material will withstand without permanent or *plastic* deformation. If the specimen is loaded further, the curve becomes increasingly less linear. If a specimen is loaded to point X in Figure D-17(c), and then unloaded, the resulting unloading curve XX_1 is linear and essentially parallel to the original elastic curve. The horizontal separation between the bases of the two curves is the *permanent set* or *plastic strain* corresponding to the stress at X. The elastic limit cannot be determined without frequently unloading the specimen during the test, but it is very near the proportional limit; the proportional limit is customarily taken as the elastic limit and called the *proportional elastic limit* (PEL). Shortly after the proportional limit, ferrous metals and certain other materials exhibit a well-defined, "sharp-kneed" *yield* point—a stress where there is a marked increase in strain without an increase in stress as shown in Figure D-17(b). The corresponding stress is called the *yield stress* or *yield strength* (σ_y). For materials without well-defined yield points, and sometimes for those with yield points, an arbitrary yield strength is defined as the stress creating a specified permanent set, often 0.2 percent of original length.

The *ultimate tensile strength* (σ_u or UTS) is the maximum load sustained by the specimen (the highest point on the stress-strain curve) divided by the *original* cross-sectional area, and as such, is a conservative measure of the specimen's strength. The *reduction in area* or *necking down* is the contraction in cross section at the fracture, expressed as a percentage of the original area. The *fracture* or *rupture* stress is the failure load divided by the reduced area.

D-6.3.2 Compressive Stress-Strain Diagram. The compressive stress-strain curve is similar to the tensile curve up to the yield point. Thereafter, increasing specimen cross section (rather than decreasing, as in the tensile test) causes the curve to diverge from the tensile curve. *Compressive yield* strength is defined as the maximum compressive stress that a ductile material can withstand without a predefined amount of deformation. *Ultimate* strength is the maximum compressive stress that a material can withstand without fracture. Some ductile materials will not fail in a compression test. If a specimen is first plastically strained in tension, yield stress in compression is reduced and vice versa.

D-6.3.3 Relationship Between Strength and Loading. Materials that yield more than 5 percent before fracture are classed as *ductile*. Relatively definite relationships exist between the strength of ductile materials in tension and their strength in compression, shear, and bearing. Compressive strength is approximately equal to tensile strength. Shear yield strength is normally taken as two-thirds tensile yield, although it may be as low as one-half to five-eighths tensile yield. Bearing yield ranges from 0.9 to 1.5 times tensile yield, depending on the application. Materials that yield less than 0.5 percent before fracture are classified as brittle. Brittle materials, such as concrete, cast iron, ceramics, polymers, etc., are usually much stronger in compression than tension and fail by fracture rather than yield.

D-6.4 Hardness. Hardness is variously defined as resistance to local penetration, scratching, abrasion, or to yielding. The resistance to local penetration, or *indentation hardness*, is used widely as a measure of hardness, and indirectly as an indicator of other properties, including strength. Indentation hardness is measured on several scales by specialized equipment.

Brinell hardness is determined by forcing a hardened sphere under known load into the surface of the material, and measuring the diameter of the resulting indentation. The Brinell hardness number is the load used in kilograms, divided by the surface area of the indentation in square millimeters.

Rockwell hardness is indicated by the depth of penetration of an indenter. The indenter is either a steel ball of specified diameter or a *Brale*—a spherical-tipped diamond cone of 120 degree included angle and 0.2-millimeter tip radius. A minor load of 10 kilograms is applied to initiate penetration and hold the indenter in place. A 60-, 100-, or 150-kilogram major load is then applied and released. Penetration is read from an indicator dial with the minor load still on the indenter. Hardness is expressed as a number equal to a constant less the number of gage units of penetration; harder materials will have higher hardness numbers. The dial on Rockwell hardness indicators is arranged to read hardness directly. A variety of combinations of indenter and major load are possible; the most commonly used are *Rockwell B* (R_B) with a 1/16-inch steel ball indenter and 100-kilogram major load, and *Rockwell C* (R_C) with a Brale indenter and 150-kilogram major load.

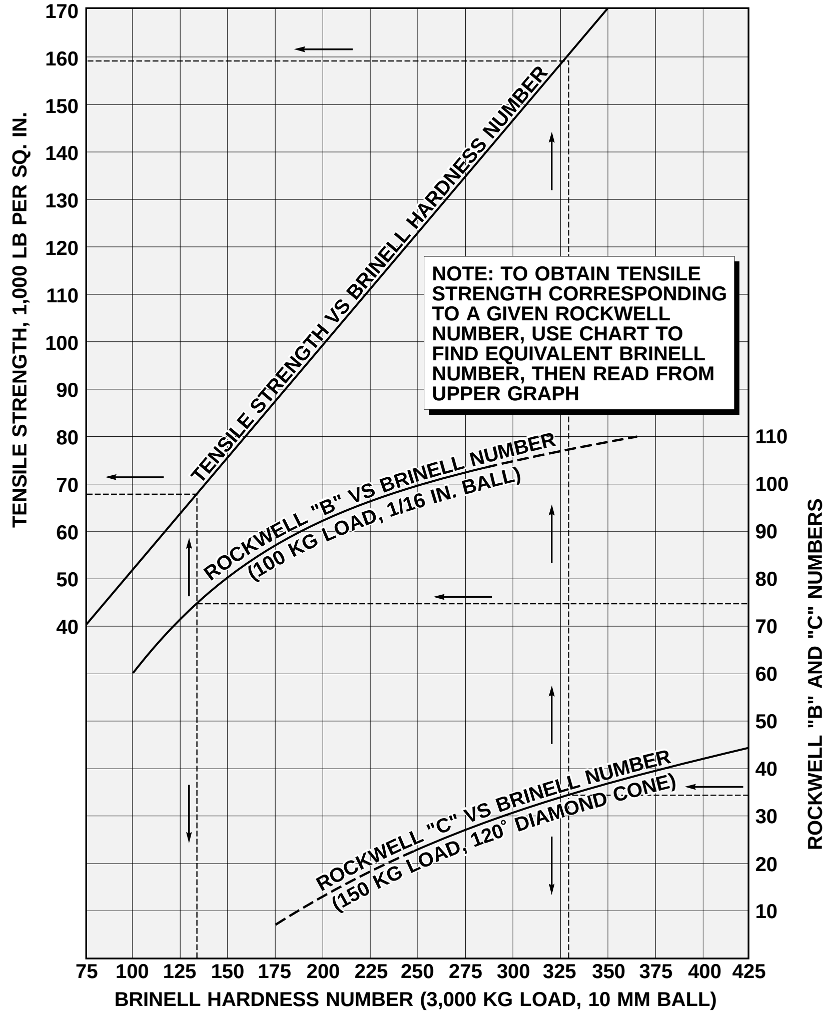

Figure D-18. Steel Tensile Strength vs Hardness.

There is a more or less definite relationship between hardness and tensile strength for any material. Once the relationship has been determined experimentally, the strength of a sample of the material can be estimated by the relatively simple Brinell or Rockwell tests. Figure D-18 shows the relationship between tensile strength and hardness for steel; ultimate tensile strength is approximately 500 times the Brinnel hardness number.

D-6.5 Additional Definitions. The following terms are frequently encountered in discussions of material strength and structural applications:

Carbon steel. Carbon steel owes its properties chiefly to the presence of carbon, without substantial amounts of other alloying elements. It is also termed *ordinary steel*, *straight carbon steel*, and *plain carbon steel*.

Case hardening. A process of hardening a ferrous allow so that the surface layer, or *case*, is made substantially harder than the interior or *core*. Typical case-hardening processes are *carburizing* and *quenching*, *cyaniding*, *carbonitriding*, *nitriding*, *induction hardening*, and *flame hardening*.

Charpy Test. A pendulum type of impact test in which a specimen, supported at both ends as a simple beam, is broken by the impact of the falling pendulum. The energy absorbed in breaking the specimen, as determined by the decreased rise of the pendulum, is a measure of the impact strength, or *toughness*, of the metal.

Cold work. Plastic deformation at such temperatures and rates that substantial increases occur in the strength and hardness of the metal. Visible structural changes include changes in grain shape and, in some instances, mechanical twinning or banding.

Cooling stresses. Stresses developed by uneven contraction or external constraint of metal during cooling; also those stresses resulting from localized plastic deformation during cooling.

Corrosion fatigue. The repeated cyclic stressing of a metal in a corrosive medium, resulting in more rapid deterioration of properties than would be encountered as a result of either cyclic stressing or of corrosion alone.

Creep. The flow or plastic deformation of metals held for long periods of time at stresses lower than the normal yield strength. The effect is particularly important if the temperature of stressing is in the vicinity of the recrystallization temperature of the metal.

Creep limit. The maximum stress that will result in creep at a rate lower than an assigned rate.

Endurance limit. The maximum stress that a metal will withstand without failure during a specified large number of cycles of stress. If the term is employed without qualification, the cycles of stress are usually such as to produce complete reversal of flexural stress.

Endurance ratio. The ratio of the endurance limit for cycles of reversed flexural stress to the tensile strength.

Fatigue. The tendency for a metal to break under conditions of repeated cyclic stressing considerably below the ultimate tensile strength.

Fatigue crack or failure. A fracture starting from a nucleus where there is an abnormal concentration of cyclic stress and propagating through the metal. The surface is smooth and frequently shows concentric (sea shell) markings with a nucleus as a center.

Flow stress. The shear stress required to cause plastic deformation of metals.

Hot working. Plastic deformation of metal at such a temperature and rate that strain hardening does not occur. The lower temperature limit for this process is the recrystallization temperature.

Impact Test. A test to determine the energy absorbed in fracturing a test bar at high velocity. The test may be in tension or in bending. A *notch* test is made with a notched sample, to test resistance to multiaxial stresses and stress concentration effects.

Malleability. The ease with which a metal deforms when subjected to rolling or hammering.

Modulus of Rigidity. In a torsion test, the ratio of the unit shear stress to angular displacement per unit length in the elastic range. Modulus of rigidity corresponds to the modulus of elasticity in the tension test.

Modulus of Rupture. The ultimate strength of the breaking load per unit area of a specimen tested in torsion or in bending (flexure). In tension, modulus of rupture is the tensile strength.

Notch brittleness. Susceptibility of a material to brittleness in areas containing a groove, scratch, sharp fillet, or notch.

Notch fatigue factor. The reduction caused in fatigue strength by the presence of a sharp notch in the stressed test section.

Notch sensitivity. The reduction in nominal strength caused by the presence of a stress concentration, usually expressed as the ratio of the notched to the unnotched strength.

Operating stress. The stress to which a structural unit is subjected during service.

Plasticity. The ability of a metal to be deformed extensively without rupture.

Proof load. The test load applied to anchors, chains, or other parts, fittings, or structure to demonstrate proper design and construction and satisfactory material.

Proof strength. The strength of a material, part, or structure as established by a proof test.

Proof stress. In a test, stress that will cause a specified permanent deformation in a material, usually 0.01 percent or less.

Residual stress. Stresses set up within a metal by nonuniform plastic deformation. This deformation may be caused by cold working or by drastic gradients of temperature from quenching or welding.

Resilience. The tendency of a material to return to its original shape after the removal of a stress that has produced elastic strain.

Shear Modulus. Modulus of rigidity.

Strain hardening. An increase in hardness and strength caused by plastic deformation at temperatures lower than the recrystallization range.

Tangent modulus. The slope of the stress-strain curve of a metal at any point along the curve in the plastic region. In the elastic region, the tangent modulus is equivalent to *Young's modulus*.

Thermal stresses. Stresses in metal, resulting from nonuniform distribution of temperature.

Toughness. The ability of a material to absorb energy before fracture; usually represented by the area under a stress-strain curve, and therefore a function of both ductility and strength.

Welding stress. The stress resulting from localized heating and cooling of metal during welding.

Work hardness. Hardness developed in metal as a result of cold working.

D-6.6 Failure Modes and Safety Factors. If a structural member or part is to carry applied loads safely, a maximum permissible stress must be determined. This *allowable stress,* also called *working stress, design stress, safe stress, etc.,* is used to establish minimum component dimensions or maximum component loads. Allowable stress is found by dividing the applicable material property—yield strength, ultimate strength, fatigue strength—by an appropriate factor of safety. The factor of safety should be chosen only after all other factors contributing to or detracting from the reliability of the member have been quantified as thoroughly as possible. These factors include assumptions implicit in the structural analysis and uncertainties as to the magnitude and kind of operating loads, reliability of the materials used, operating environment, level of quality control that can be implemented during fabrication and installation, and level of knowledge about possible failure modes. An additional important consideration is the potential damage should the component or system fail, particularly when there is danger to human life.

In general, the ductility of the material and type of loading specify the failure mode and the property to which the factor of safety should be applied to determine allowable stress. There are three general cases:

- Brittle materials,
- Ductile materials in static loading, and
- Ductile materials in cyclic loading.

D-6.6.1 Brittle Materials. For brittle materials in uniaxial stress, the factor of safety (FS) is applied to ultimate strength (σ_u) to determine allowable stress:

$$\sigma_{allow} = \frac{\sigma_u}{FS}$$

For brittle materials in biaxial stress, the *maximum normal stress theory* predicts failure of brittle materials under static loading if the compressive principal stress is greater than the ultimate compressive strength, or the tensile principle stress is greater than the ultimate tensile strength. The principle stresses, σ_1, σ_2, are determined as described in Paragraph 2-8.2. By plotting compressive stresses as negative and tensile stresses as positive on σ_1-σ_2 coordinates, a *safe stress combination envelope* can be defined as a rectangle bounded by the ultimate compressive and tensile principle stresses, as shown in Figure D-19. An allowable stress envelope is created by applying a safety factor to the ultimate compressive and tensile stresses to define a smaller rectangle.

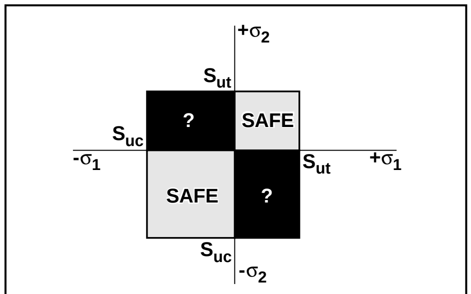

Figure D-19. Safe Stress Combinations for Brittle Materials (Maximum Normal Stress Theory).

Experimental evidence shows that failures occur in the second and fourth quadrants, even though the stresses are less than the ultimate strengths. The *Coulomb-Mohr theory* modifies the failure line in the second and fourth quadrants, shown in Figure D-20, along with typical failure data.

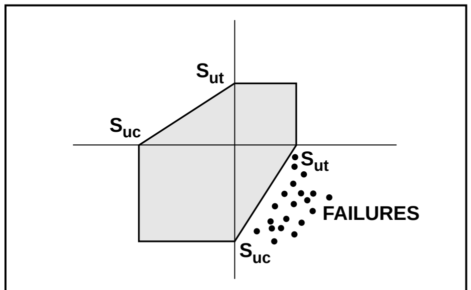

Figure D-20. Safe Stress Combinations for Brittle Materials (Coulomb-Mohr Theory).

D-6.6.2 Ductile Materials in Static Loading. Plastic deformation is unacceptable for most applications, so yield is considered failure. Allowable stresses for ductile materials in uniaxial stress are found by applying the factor of safety (FS) to ultimate tensile strength (σ_u):

$$\sigma_{allow} = \frac{\sigma_u}{FS}$$

Compressive and tensile yield strengths are equal for ductile materials.

For ductile materials in biaxial stress or pure shear, the *maximum shear stress theory* predicts that yield will begin when maximum shear stress equals the shear yield strength. Shear yield strength (τ_y) is 60 to 65 percent of tensile yield strength for ductile materials, but is assumed to be one-half tensile yield strength by the theory. Maximum shear stress is equal to:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

The basic criteria is to keep maximum shear stress below one-half the tensile yield strength, producing the safe stress envelope shown in Figure D-21. The shape of the envelope is similar to that of the Coulomb-Mohr theory for brittle materials, but is based on yield strength rather than ultimate strength. The envelope is symmetrical because tensile and compressive yield strength are assumed equal for ductile materials. The factor of safety is defined as:

$$\mathrm{FS} = \frac{\tau_{\mathrm{allow}}}{\tau_{\mathrm{max}}} = \frac{\sigma_y}{2\tau_{\mathrm{max}}}$$

Allowable shear stress is then:

$$\tau_{\mathrm{allow}} = \frac{\sigma_y}{2\mathrm{FS}}$$

The factor of safety is incorporated into the allowable stress diagram by setting the σ_1 and σ_2 intercepts equal to τ_{allow}.

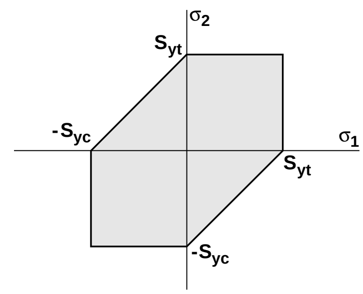

Figure D-21. Safe Stress Combinations for Ductile Materials (Maximum Shear Stress Theory).

D-6.6.3 Ductile Materials in Cyclic Loading. Fatigue failure is failure of a component subject to cyclic loading at stresses below the yield limit. The fatigue strength is the maximum completely reversing stress a material can withstand without failing. A logarithmic plot of fatigue strength against the number of load cycles (S-N curve, shown in Figure D-22) shows a linear relationship in the region between 1,000 and 1,000,000 cycles. For fewer than 1,000 cycles, fatigue strength is equal to ultimate strength; after 1,000,000 cycles there is no further strength reduction and the curve is flat. The maximum stress for an infinite life is the *endurance strength* (s_e). Endurance strengths for steel and cast iron are:

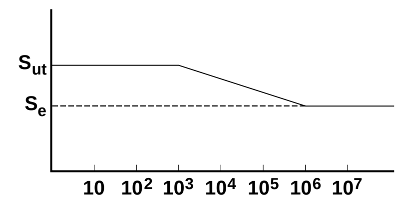

Figure D-22. Typical S-N Curve for Steel.

Steel	s_e	=	$0.5s_u$	(s_u < 200,000 psi)
		=	100,000 psi	(s_u > 200,000 psi)
Cast iron	s_e	=	$0.4s_u$	

where:

s_u = ultimate strength for the type stress (i.e., tensile, compressive, shear)

Fatigue strength of aluminum never levels off, but continues to decrease as the number of cycles increase. Endurance strength for aluminum is taken as the fatigue strength at 100,000,000 cycles and is approximately:

Cast	s_e	=	$0.3s_u$
Wrought	s_e	=	$0.4s_u$

An *S-N* curve can be used to establish limiting loads for an anticipated number of cycles, or to predict the approximate number of cycles to failure for known stress levels.

Fluctuating stresses are created in a material when:

- A load is intermittently applied and released in one direction only.
- A component is subject to both a static load and a cyclic (reversing) load that is not great enough to cancel the static load and reverse the stresses in the component.
- An applied load varies between upper and lower limits, but does not reverse.

The mean stress is:

$$s_{\mathrm{mean}} = \frac{s_{\mathrm{max}} + s_{\mathrm{min}}}{2}$$

The alternating stress is half the stress range:

$$s_{\mathrm{alt}} = \frac{s_{\mathrm{max}} - s_{\mathrm{min}}}{2}$$

Failure stress of a material under fluctuating stress is a function of both yield strength and endurance strength. The two criteria are related by plotting s_{alt} on a vertical scale, and s_{mean} on a horizontal scale, as shown in Figure D-23. A failure line (*Soderberg line*) is drawn from the endurance strength (s_e) on the vertical scale, and yield strength (s_y) on the horizontal. The enclosed triangle defines acceptable combinations of alternating and mean stress. Factors of safety can be applied to s_e and s_y to define a safe stress line, as shown.

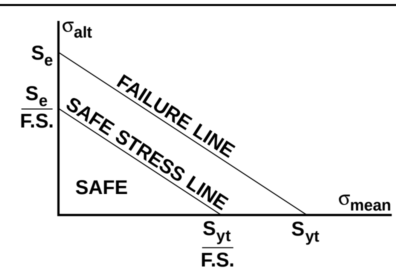

Figure D-23. Safe Operating Stress Envelope Determined by Soderberg Line.